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Simulation of Technical Systems on the basis of Vector Optimization (1. Equivalent Criteria)

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Abstracts

Presents a new methodology for modeling technical systems (TS), a model of which is represented by a vector problem of mathematical programming (VPMP). The model is designed for evaluation and selection of necessary tech-nical specifications (criteria) and to identify the relevant parameters of the TS. A theoretical study and construct a mathematical model of the technical system as a vector programming problem whose solution is based on the normaliza-tion criteria principle guaranteed result. Problem is solved with equivalent cri-teria. Modeling methodology is illustrated by a numerical example of a model of the TS, in the form of a vector problem of nonlinear programming is imple-mented in Matlab.

Keywords:

Introduction

Progress in the development of most branches of industry connected with the creation of new technical (engineering) systems (TS), which meets the latest achievements of science and technology. Created TS during operation must continually modernize, and, as they become obsolete, replaced by a more ad-vanced products. These problems put before design organizations the need to accelerate and increase the volume of design works on creation of new TS.

Issues to accelerate the design of TS, improvement of their quality, reliability stimulated: creation of mathematical models adequately describing the operation of the technical system; continuous improvement of systems of processing of the information related to the design of TS. Therefore, the problem of mathematical modeling of technical systems, as an integral part of computer-aided design (CAD), great attention is paid in Russia [1-5] and abroad in theoretical [6-7] and applied aspects [8-9].

In the study and modeling of new technical objects, systems, a model of which is represented by a vector problem of mathematical programming [10], it is necessary to give estimates of results of modeling and making optimal decisions based on them [10, 11]. This raises the problem of assessing results as the equivalent criteria, and in particular importance (priority) criterion. On the solution of these problems directed this work.

The purpose of this work – theoretical justification, methodology of constructing models and mathematical modeling of technical system in the form of the vector problem of mathematical programming (VPMP), and also its decision at equivalent criteria.

For the realization of this goal it is shown the model of technical system, which is represented by a vector problem of mathematical programming. Theoretical substantiation and building of algorithm of the decision of VPMP based on normalization of the criteria, the principle guaranteed result. Modeling methodology is illustrated by a numerical example of a model TS, in the form of a vector problem of nonlinear programming is implemented in Matlab [12].

Mathematical model of the technical system

The problem of choosing optimal parameters of technical systems on the functional characteristics associated with the release of high quality products. This problem always arises in the study, analysis and design of technical systems in CAD.

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где x_j^{min}, x_j^{max}, $\forall j \in \boldsymbol{N}$ - нижний и верхний пределы изменения вектора параметров технической системы.

Considered a technical system, the functioning of which depends on the $\boldsymbol{N}$ - structural parameters set[1] $X=\{x_1\ x_2 \dots x_N\}$, where N - number of parameters, each of which lies within specified limits:

$$x_j^{min} \leq x_j \leq x_j^{max},\ j=\overline{1,N},\ \text{or}\ X^{min} \leq X \leq X^{max}, \qquad (1)$$

where x_j^{min}, x_j^{max}, $\forall j \in \boldsymbol{N}$ - lower and upper limits of variation of the parameter vector of the technical system

Result functioning technical system defined by the set $\boldsymbol{K}$ To technical characteristics of $f_k(X)$, $k=\overline{1,K}$, which are functionally dependent on the design parameters of the TS $X=\{x_j,\ j=\overline{1,N}\}$, together they represent a vector function:

$$F(X) = (f_1(X)\ f_2(X) \dots f_K(X))^{\mathrm{T}}. \qquad (2)$$

Set of characteristics (criteria) is divided into two subsets $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$: $\boldsymbol{K}=\boldsymbol{K}_1 \cup \boldsymbol{K}_2$.

$\boldsymbol{K}_1$ - a subset of the technical characteristics, the numerical values which it is desired to receive as it is possible above:

$$f_k(X) \rightarrow \max,\ k=\overline{1,K_1}.$$

$\boldsymbol{K}_2$ - is a subset of the technical characteristics, the numerical values which it is desirable to receive as it is possible below:

$$f_k(X) \rightarrow \min,\ k=\overline{K_1+1,K},\ \boldsymbol{K}_2 \equiv \overline{K_1+1,K}$$

Mathematical model of a technical system solving as a whole a problem of a choice of optimal design solutions (choice of the TS optimum parameters) can be represented as a vector problem of mathematical programming:

$$Opt\ F(X) = \{max\ F_1(X) = \{max\ f_k(X),\ k = \overline{1,K}_1\}, \qquad (3)$$

$$min\ F_2(X) = \{min\ f_k(X),\ k = \overline{1,K}_2\}\}, \qquad (4)$$

$$G(X) \leq 0, \qquad (5)$$

$$x_j^{min} \leq x_j \leq x_j^{max},\ j = \overline{1,N}, \qquad (6)$$

where X - a vector of controlled variables (design data of the TS) from (1); $F(X)=\{f_k(X),\ k=\overline{1,K}\}$ - the vector criterion which everyone component submits the TS characteristic from (2), functionally depending on a vector of variables X; in (5) $G(X)=(g_1(X)\ g_2(X) \dots g_M(X))^{\mathrm{T}}$ – a vector function of the constraints imposed on functioning of the TS.
They are determined going in it technological, physical, and similar processes and can be presented functional constraints, for example, $f_k^{min} \leq f_k(X) \leq f_k^{max}$, $k=\overline{1,K}$.

It is supposed that the functions $f_k(X)$, $k=\overline{1,K}$ are differentiated and convex, $g_i(X)$, $i=\overline{1,M}$ continuous and given constraints (5) - (6) set of admissible points S isn't empty and represents a compact:

$$\boldsymbol{S}=\{X \in \boldsymbol{R}^N \mid G(X) \leq 0,\ X^{min} \leq X \leq X^{max}\} \neq \emptyset. \qquad (7)$$

Relations (3)-(6) form a mathematical model of the TS. It is required to find such vector of the $X^o \in \boldsymbol{S}$ parameters at which everyone a component the vector - function the $F_1(X)=\{f_k(X),\ k=\overline{1,K}_1\}$ accepts the greatest possible value, and the vector - function $F_2(X)=\{f_k(X),\ k=\overline{1,K}_2\}$ accepts a minimum value.

To solve this class VPMP in this article uses the methods based on the principle of normalization criteria and the principle of guaranteed result [10, 11]. They allow you to decide when VPMP equivalent criteria and priority for a given criterion. In this article the main attention is paid to TS modeling at equivalent criteria.

[1] Another way to write the vector $X=\{x_j,\ j=\overline{1,N}\}$.

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Decision vector problems with equivalent criteria

Theoretical Foundations of solutions of vector optimization problems

For development of methods of the solution of problems of vector optimization we will enter definitions:

- relative estimate;
- equivalence of criteria in VPMP (axiom);
- relative level for all criteria;
- the principle of an optimality of the solution of problems of vector optimization at equivalent criteria, and related theorems. For more details see [10, 11].

Definition 1. Criteria in VPMP (3)-(6) are normalized if the following equality is carried out:

$$\lambda_k(X) = \frac{f_k(X) - f_k^0}{f_k^* - f_k^0}, \ \forall k \in \boldsymbol{K}, \qquad (8)$$

where $\lambda_k(X)$, $\forall k\in\boldsymbol{K}$ is the relative estimate of a point $X\in S$ k-th criterion; $f_k(X)$ - k-th criterion at the point $X\in\boldsymbol{S}$; f_k^* - value of the k-th criterion at the point of optimum X_k^*, obtained in VPMP (3)-(6) of individual k-th criterion; f_k^0 is the worst value of the k-th criterion (antioptimum) at the point X_k^0 (Superscript 0 - zero) on the admissible set $\boldsymbol{S}$ in VPMP (3)-(6); the task at *max* (3), (5), (6) the value of f_k^0 is the lowest value of the k-th criterion $f_k^0 = \min_{X\in S} f_k(X)$ $\forall k\in\boldsymbol{K_1}$ and task *min* f_k^0 is the greatest: $f_k^0 = \max_{X\in S} f_k(X)$ $\forall k\in\boldsymbol{K_2}$.

The relative estimate of the $\lambda_k(X)$, $\forall k\in\boldsymbol{K}$ is first, measured in relative units; secondly, the relative assessment of the $\lambda_k(X)$ $\forall k\in\boldsymbol{K}$ on the admissible set is changed from zero in a point of X_k^0 :

$\forall k\in\boldsymbol{K}$ $\lim_{X\to X_k^o} \lambda_k(X)=0$,

to the unit at the point of an optimum of X_k^* :

$\forall k\in\boldsymbol{K}$ $\lim_{X\to X_k^*} \lambda_k(X)=1$ i.e.:

$\forall k\in\boldsymbol{K}$ $0\le\lambda_k(X)\le 1$, $X\in S$ - (9)

this allows the comparison criteria, measured in relative units, among themselves by joint optimization.

Definition 2 **(Axiom 1.** About equality and equivalence of criteria in an admissible point of VPMP)

In VPMP two criteria with the indexes $k\in\boldsymbol{K}$, $q\in\boldsymbol{K}$ shall be considered as equal in $X\in\boldsymbol{S}$ point if relative estimates on k-th and q-th to criterion are equal among themselves in this point, i.e. $\lambda_k(X) = \lambda_q(X)$, $k, q \in \boldsymbol{K}$.

We will consider criteria equivalent in VPMP if in $X\in\boldsymbol{S}$ point when comparing in the numerical size of relative estimates of $\lambda_k(X)$, $k=\overline{1,K}$, among themselves, on each criterion of $f_k(X)$, $k=\overline{1,K}$, and, respectively, relative estimates of $\lambda_k(X)$, isn't imposed conditions about priorities of criteria.

Definition 3. The relative λ level in VPMP is the bottom assessment of a point of $X\in\boldsymbol{S}$ among all relative estimates of $\lambda_k(X)$, $k=\overline{1,K}$, i.e. λ an essence bottom bending around the $\lambda_k(X)$ functions:

$\forall X\in\boldsymbol{S}$ $\lambda \le \lambda_k(X)$, $k=\overline{1,K}$, (10)

the bottom level for performance of a condition (10) in an admissible point is defined by a formula

$\forall X\in\boldsymbol{S}$ $\lambda = \min_{k\in K} \lambda_k(X)$. (11)

The relations (10) and (11) are interrelated and are moving further from operations define min(max) to the restrictions and vice versa.

Introduction level λ which allows to combine all the criteria VPMP one numeric characteristics and produce over her certain operations, thus doing these operations on all the criteria, measured in relative units. The level λ is

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functionally depends on the $X \in \boldsymbol{S}$ - changing it, you can change and what λ. From here can be formulated and the rule of finding the optimal solution.

Definition 4. (*Principle of an optimality*).

The vector problem of mathematical programming at equivalent criteria is solved, if the point of $X^o \in S$ and a maximum level of λ^o (the top index o - optimum) among all relative estimates such that is found

$$\lambda^o = \max_{X \in S} \min_{k \in K} \lambda_k(X). \quad (12)$$

Using interrelation of expressions (10) and (11), we will transform a maximine problem (12) to an extreme problem

$$\lambda^o = \max_{X \in S} \lambda, \quad (13)$$

$$\lambda \le \lambda_k(X),\ k=\overline{1,K}. \quad (14)$$

The resulting problem (13)-(14) let's call the λ-problem.

λ-problem (13)-(14) has $(N+1)$ dimension, as a consequence of the result of the solution of λ-problem (13)-(14) represents an optimum vector of $\boldsymbol{X^o} \in \boldsymbol{R}^{N+1}$, (N+1) which component an essence of the value of the λ^o, i.e. $\boldsymbol{X^o}=\{x_1^o, x_2^o, ..., x_N^o, x_{N+1}^o\}$, thus $x_{N+1}^o = \lambda^o$, and $(N+1)$ a component of a vector of $\boldsymbol{X^o}$ selected in view of its specificity.

The received a pair of $\{\lambda^o, X^o\}=\boldsymbol{X^o}$ characterizes the optimum solution of λ-problem (13)-(14) and according to VPMP (3)-(6) with the equivalent criteria, solved on the basis of normalization of criteria and the principle of the guaranteed result. We will call in the optimum solution of $\boldsymbol{X^o}=\{X^o, \lambda^o\}$, X^o - an optimal point, and λ^o - a maximum level.

Theorem 1. (The theorem of the most contradiction criteria in VPMP with equivalent criteria).

In convex VPMP at the equivalent criteria, solved on the basis of normalization of criteria and the principle of the guaranteed result, in an optimum point of $\boldsymbol{X^o}=\{\lambda^o, X^o\}$ there is always two criteria - denote their indexes $q \in \boldsymbol{K}$, $p \in \boldsymbol{K}$ (which in a sense are the most contradiction of the criteria $k = \overline{1,K}$), for which the equality:

$$\lambda^o = \lambda_q(X^o) = \lambda_p(X^o),\ q, p \in \boldsymbol{K},\ X \in S, \quad (15)$$

and other criteria are defined by inequalities:

$$\lambda^o \le \lambda_k(X^o)\ \ \forall k \in \boldsymbol{K},\ q \ne p \ne k. \quad (16)$$

Theorem 2. (The theorem about Pareto optimality of the solutions of VZMP at equivalent criteria).

In a convex vector problem of mathematical programming at equivalent criteria a point of an optimum X^o, received on the basis of normalization of criteria and the principle of guaranteed result, is optimum across Pareto, and if criteria are continuously differentiated, such point only one. [10, 11].

Algorithm for solving of vector optimization problems with equivalent criteria

The algorithm of the solution of VPMP (3)-(6) with equivalent criteria is executed according to the principle of an optimality and presented in the form of a number of steps.

Step 1. The problem (3)-(6) by each criterion separately is solved, i.e. for $\forall k \in K_1$ is solved at the maximum, and for $\forall k \in K_2$ is solved at a minimum. As a result of the decision we will receive:

X_k^* - an optimum point by the corresponding criterion, $k=\overline{1,K}$;

$f_k^* = f_k(X_k^*)$ – the criterion size k-th in this point, $k=\overline{1,K}$.

Step 2. We define the worst value of each criterion on $\boldsymbol{S}$: f_k^0, $k=\overline{1,K}$. For what the problem (3)-(6) for each criterion of $k=\overline{1,K_1}$ on a minimum is solved:

$f_k^0 = \min f_k(X),\ G(X) \le B,\ X \ge 0,\ k=\overline{1,K_1}$.

The problem (3)-(6) for each criterion on a maximum is solved:

$f_k^0 = \max f_k(X),\ G(X) \le B,\ X \ge 0,\ k=\overline{1,K_2}$.

As a result of the decision we will receive: $X_k^0=\{x_j, j=\overline{1,N}\}$ - an optimum point by the corresponding criterion, $k=\overline{1,K}$; $f_k^0 = f_k(X_k^o)$ – the criterion size k-th a point, X_k^0, $k=\overline{1,K}$.

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Step 3. The analysis of a set of points, optimum across Pareto, for this purpose in optimum points of $X^* = \{X_k^*$, $k=\overline{1,K}$ } are defined sizes of criterion functions of $F(X^*)=\{f_q(X_k^*)$, $q=\overline{1,K}$, $k=\overline{1,K}$ } and relative estimates

$\lambda(X^*) = \{\lambda_q(X_k^*)$, $q=\overline{1,K}$, $k=\overline{1,K}$ }, $\lambda_k(X) = \dfrac{f_k(X) - f_k^o}{f_k^* - f_k^o}$, $\forall k \in \boldsymbol{K}$:

$$F(X^*)=\begin{vmatrix} f_1(X_1^*),..., f_k(X_1^*), \\ ... \\ f_1(X_k^*),..., f_k(X_k^*) \end{vmatrix}, \lambda(X^*)=\begin{vmatrix} \lambda_1(X_1^*),..., \lambda_k(X_1^*), \\ ... \\ \lambda_1(X_k^*),..., \lambda_k(X_k^*) \end{vmatrix}. \quad (17)$$

As a whole on a problem of accordance with (9)$\forall k \in \boldsymbol{K}$ the relative assessment of $\lambda_k(X)$, $k=\overline{1,K}$ lies within $0 \le \lambda_k(X) \le 1$, $\forall k \in \boldsymbol{K}$.

Step 4. Creation of the λ-problem.

Creation of λ-problem is carried out in two stages: initially built the maximine problem of optimization with the normalized criteria which at the second stage will be transformed to the standard problem of mathematical programming called λ-problem.

For construction maximine a problem of optimization we use definition - relative level (11) $\forall X \in \boldsymbol{S}$ $\lambda = \min_{k \in K} \lambda_k(X)$.

The bottom λ level is maximized on $X \in S$, as a result we will receive a maximine problem of optimization with the normalized criteria.

$$\lambda^o = \max_x \min_k \lambda_k(X),\ G(X) \le B,\ X \ge 0. \quad (18)$$

At the second stage, using interrelation (10) and (11), we will transform a problem (18) to a standard problem of mathematical programming:

$$\lambda^o = \max \lambda, \qquad \lambda^o = \max \lambda, \quad (19)$$

$$\lambda - \lambda_k(X) \le 0,\ k = \overline{1,K}, \quad \rightarrow \quad \lambda - \frac{f_k(X) - f_k^o}{f_k^* - f_k^o} \le 0,\ k = \overline{1,K}, \quad (20)$$

$$G(X) \le B,\ X \ge 0, \qquad G(X) \le B,\ X \ge 0, \quad (21)$$

where the vector of unknown of X has dimension of N+1: $X=\{\lambda, x_1, \dots, x_N\}$.

Step 5. Solution of λ-problem.

λ-problem (19)-(21) is a standard problem of convex programming and for its decision standard methods are used.

As a result of the solution of λ-problem it is received:

$\boldsymbol{X^o}=\{\lambda^o, X^o\}$ - an optimum point;

$f_k(X^o)$, $k=\overline{1,K}$ - values of the criteria in this point;

$\lambda_k(X^o) = \dfrac{f_k(X^o) - f_k^o}{f_k^* - f_k^o}$, $k=\overline{1,K}$ - sizes of relative estimates;

λ^o - the maximum relative estimates which is the maximum bottom level for all relative estimates of $\lambda_k(X^o)$, or the guaranteed result in relative units, λ^o guarantees that all relative estimates of $\lambda_k(X^o)$ more or are equal λ^o in X^o point to λ^o, $\lambda_k(X^o) \ge \lambda^o$, $k=\overline{1,K}$ or $\lambda^o \le \lambda_k(X^o)$, $k=\overline{1,K}$, $X^o \in S$, (22)

and according to the theorem the 2 point of $\boldsymbol{X^o}=\{\lambda^o, x_1, \dots, x_N\}$ is optimum across Pareto.

Technology of research, creation of mathematical model of technical system and decision-making

The methodology of process of construction of the TS mathematical model and decision-making on its basis is intended for the analysis and TS synthesis at a design stage and operation. The main attention is paid, first, to creation of the TS model and methods of the solution of VPMP, secondly, a place of these models and methods in problems of design of technical (engineering) systems. The flowchart of methodology is submitted in fig. 1 and described, how sequence of a number of steps (blocks).

Block 0. The specification on developed products where the purposes and requirements to technical systems are formulated is formed.

Block 1. For research of the physical processes proceeding in the TS, and creation of mathematical models of such processes fundamental laws of physics are used: modeling of magnetic, temperature fields; conservation laws of energy, movement, etc.

At the same stage for the solution of the problems which are cornerstone of studied physical processes, numerical methods are developed. As a rule, modeling and calculations are carried out by means of the software developed for this purpose and computer facilities. Then the software is tested for adequacy to real physical data.

If the main physical processes proceeding in the TS are known, and functional dependence of each characteristic and restrictions on the TS parameters is known further, such situation is called "modeling in the conditions of definiteness". If physical processes in the TS are insufficiently studied, such situation is called "modeling in the conditions of uncertainty". In this case, construction of experimental (regression) models associated with the analysis of input and output data, see [5].

Block 2. The full list of all functional characteristics of technical systems and parameters on which these characteristics depend is formed. Their verbal description is given.

Block 3. The technical and information interrelation of all TS components is established, i.e. the structure is under construction. Here the problem of a choice of the best (in any sense) TS structures is solved, i.e. the problem of structural optimization [3] is carried out.

Block 4. TS mathematical model formation.

It includes four stages.

Definition of the purposes and indicators of functioning of the TS.

Quality of functioning of the TS by any is defined by a set of technical (output) characteristics which represent a quantitative measure of reflection of requirements to TS properties. For electronic schemes such characteristics are: output power, speed, accuracy assessment, dimensions, etc. For engines - the output power, speed, efficiency, etc. We will designate set of all vector characteristics a set " $\boldsymbol{K}$", and an index $k = \overline{1, K}$.

Identification of a vector of the TS variables.

The technical system in a statics is investigated. Major factors and parameters which remain constants for the studied period of time are identified. The variable parameters which size it is desirable to determine and which size can change in the course of design are identified. Them also call operated parameters or design parameters. They are in turn subdivided into internal and external parameters. For electronic schemes internal parameters are: resistance, capacities, inductance, coefficients of strengthening of transistors, etc. For mechanical systems, for example, engines: combustion chamber volume, piston stroke, their quantity, etc. External parameters are the factors connected with environment (for example, temperature), power supplies, etc.

These parameters usually also are subject to definition.

We will designate a vector of design data (a vector of variables) the TS through $X = \{x_j, j = \overline{1, N}\}$, where $\boldsymbol{N}$ - a set of indexes (N- their number) variables. Limits of change of a vector of variables come to light:

$x_j^{\min} \le x_j \le x_j^{\max}$, $j=\overline{1, N}$, or $X^{min} \le X \le X^{max}$,

where $x_j^{\min}$, $x_j^{\max}$, $\forall j \in \boldsymbol{N}$, bottom and top limits of change of a vector of variables. These relations are called parametric restrictions

Formation of a vector of criteria of the TS.

The criterion is a measure of a quantitative estimation of functional dependence of a vector of variables X from each output characteristic of $k=\overline{1, K}$. The set of functional dependences of characteristics represents vector criterion:

$F(X) = \{f_k(X), k = \overline{1, K}\}$,

where $K(K)$ is the set of (number) indexes of criteria of the TS.

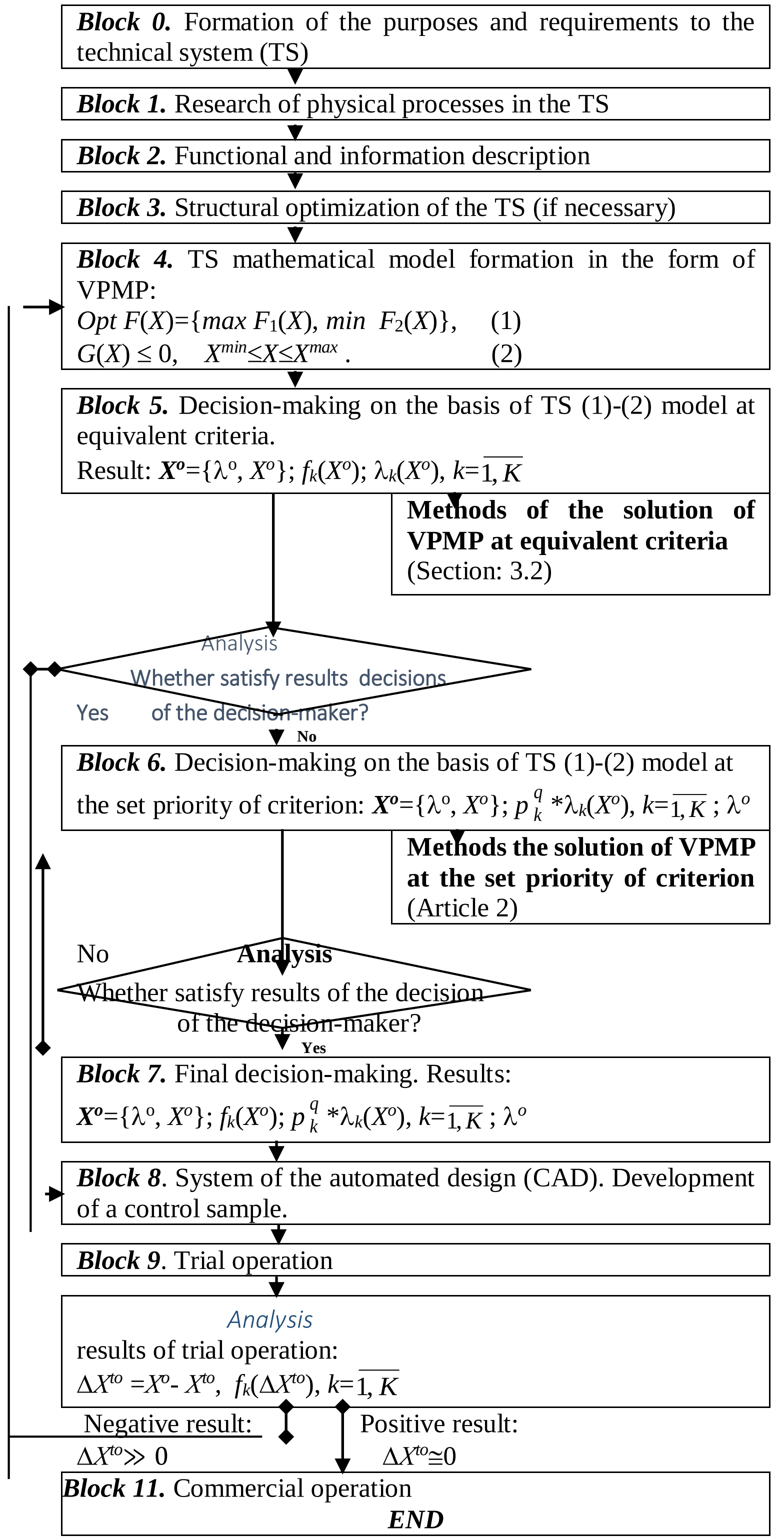

Fig. 1. The flowchart of process of design and TS mathematical model place in decision-making.

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Optimization on one of characteristics (criterion) led to deterioration of other characteristics of the TS, as a result the chosen design decision was insolvent. This circumstance also constrained wide use of methods of optimization in the analysis and TS synthesis. The solution of a question, in our opinion, is consolidated to creation of mathematical model which would be adequate to the TS, i.e. would consider all characteristics of the TS at its functioning at the same time.

The definition of functional dependencies between constraints and parameters of the TS.

Are investigated and imposed on functioning of the TS of restriction of four types:

the restrictions which are put forward by the specification on creation of the TS;

technological restrictions;

the restrictions connected with physical processes, proceeding in the TS;

restrictions on functioning the TS connected with environment.

Functional dependence of parameters among themselves, and, according to technical requirements to entrance and output parameters (6) is established: $X^{min} \leq X \leq X^{max}$ - parametrical restrictions.

Taking into account the admissible range of change of variables of restriction in a symbolical form it is possible to present in the form of inequalities (5): $G(X) \leq 0$ or $(g_1(X) \leq 0 \;\; g_2(X) \leq 0 \; \ldots \; g_M(X) \leq 0)^T$, where $M(M)$ - a set (number) of restrictions of the TS.

Let part of characteristics of $f_k(X)$, $k = \overline{1, K}_1$, $K_1 \subset K$, on quantity it is desirable to receive as much as possible (i.e. the corresponding criteria are maximized), and part of $f_k(X)$, $k = \overline{1, K}_2$, $K_2 \subset K$ are minimized. Taking into account these requirements the mathematical model solving as a whole a problem of a choice of the optimum design decision (a choice of the TS optimum parameters), it is possible to present in the form of a vector problem of mathematical programming, similarly (3)-(6) – Fig. 1, the block 4:

$$Opt\, F(X) = \{max\, F_1(X) = \{max\, f_k(X),\, k = \overline{1, K}_1\}, \quad (23)$$

$$min\, F_2(X) = \{min\, f_k(X),\, k = \overline{1, K}_2\}\}, \quad (24)$$

$$G(X) \leq 0,\;\; x_j^{\min} \leq x_j \leq x_j^{\max},\; j = \overline{1, N}. \quad (25)$$

We assume that the problem (23)-(25) belongs to the class of convex problems, and the set of admissible points of S presented by restrictions (26) isn't empty and represents a compact. From here it is possible to determine an optimum by any of criteria "*K*". VPMP (23)-(25) is the TS model in a statics, but such model can be used for research of dynamic processes for the small period of time [2].

Block 5. Decision-making on the basis of TS (23)-(25) model at equivalent criteria.

The made decision is defined by the solution of a vector problem (23)-(25) at equivalent criteria, i.e. lack of a priority on any criterion.

Methods of the decision are based on normalization of criteria and the principle of the guaranteed result are presented in section 3.2.

Result of the decision:

$$X^o = \{\lambda^o, X^o\};\; f_k(X^o),\; k = \overline{1, K};\; \lambda_k(X^o),\; k = \overline{1, K};\; \lambda^o. \quad (26)$$

(Designations in section 3.2).

The analysis of design parameters of X^o and characteristics of $f_k(X^o)$, $k = \overline{1, K}$ technical system is made. If they meet requirements of the decision-maker, we go to block 8, otherwise the next block.

Decision-making on the basis of TS (23)-(25) model at the set priority of criterion.

The decision received in the previous block is defined proceeding from equivalence of criteria of the TS. In actual practice the priority (preference) of TS any of criteria, for example, $q \in K$ is usually imposed. The decision in this case gets out of a set of points of $S_q \in S$, lying between points of X^o and X_q^*, $q \in K$.

Methods of the decision are based on normalization of criteria, the principle of the guaranteed result and axiomatics of a priority of criterion in VPMP.

The analysis of design data of X^o and characteristics of $f_k(X^o)$, $k = \overline{1, K}$ technical system with a criterion priority is made. If they meet requirements of the decision-maker, the previous block is passed to the block 7, differently.

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Block 7. Final decision-making at the set priority of criterion. Results: $X^o=\{\lambda^o, X^o\}$; $f_k(X^o)$, $k=\overline{1,K}$; $p_k^q*\lambda_k(X^o)$, $k=\overline{1,K}$; λ^o (27)

determine parameters and characteristics of technical system.

Block 8. Computer-aided design (CAD).

On the basis of design parameters of X^o by means of system of the automated design project documentation of the TS which functioning is defined by characteristics of $f_k(X^o)$, $k=\overline{1,K}$ is formed. Built a prototype that is passed into pilot operation.

Block 9. Trial operation (*to*).

Block 10. By results of trial operation experimental data of design data X^{to} and functional characteristics of $f_k(X^{to})$, $k=\overline{1,K}$ can be obtained. They are compared to the X^o parameters from mathematical model

$$\Delta X^{to}=X^o - X^{to},\ f_k(\Delta X^{to}),\ k=\overline{1,K}. \quad (28)$$

If deviations of are close to zero: $\Delta X^{to}\cong 0$, TS is put into commercial operation. If deviations are considerable: $\Delta X^{to}\gg 0$, by results of trial operation the second stage of improvement of model begins.

Block 11. Commercial operation.

Applied part of modeling of the TS it is representable in the form of methodology of the solution of VPMP with a priority of criterion and we will show on test examples of the TS models realized in Matlab system.

Adoption of the optimum decision on model of technical system

We will consider model of technical system for which are known:

• functional dependence of each characteristic (criterion) of $k\in K$ (3)-(4) on a vector of design data (variables) of $X=\{x_1, x_2\}$, the set of criteria K=4 is divided into two under sets - two criteria of max and two min;

• restrictions (5)-(6) which are functionally dependent on the same *TS* parameters.

The model of technical system is presented by a vector problem of mathematical (nonlinear) programming:

$opt\ F(X)=$

$$\{max\ F_1(X)=\{max\ f_1(X)\equiv 639+0.031x_1+0.0421x_1^2+0.039x_2^2, \quad (29)$$

$$max\ f_3(X)\equiv 402 - 0.04x_1 +0.04x_1^2 +0.08x_2^2\}, \quad (30)$$

$$min\ F_2(X)=\{min\ f_2(X)\equiv -506 +0.71x_1+528x_2 -1.9x_2^2, \quad (31)$$

$$min\ f_4(X)\equiv -803+0.203x_1+0.135x_1^2 -0.09x_1x_2\}\}, \quad (32)$$

при ограничениях at restrictions

$$10000\le f_2(X)\equiv -506 +0.71x_1 +528x_2-0.19x_2^2\le 21000, \quad (33)$$

$$10\le x_1\le 80,\ 15\le x_2\le 70. \quad (34)$$

The methodology of modeling and adoption of the optimum decision in the annex to TS (29)-(34) model at equivalent criteria, is based on axiomatics, with use of normalization of criteria and the principle a maximine, we will present in the form of sequence of steps according to section 3.2.

Step 1. The problem (29)-(34) by each criterion separately for the purpose of receiving points of private optimum X_k^* and $f_k^*=f_k(X_k^*)$, $k=\overline{1,4}$ is solved. Each of them is a nonlinear problem of optimization and for its decision in Matlab system the function fmincon(…) is used. For example, for the first criterion the problem looks as follows: (29) (33), (34), the address to her will assume an air:

[x1max,f1max]=fmincon('TehnSist_Krit1max',Xo,A,b,Aeq,beq,lb,ub,'TehnSist_Const',options)
(35)

where in parentheses input parameters are given: 'TehnSist_Krit1max' – the subprogramme of definition of function (the first criterion) and its gradient, $\frac{\partial f_k(X)}{\partial X}$, $k=\overline{1,4}$ on a Fortran; A, b, … – linear restrictions.

```
% [The subprogramme "Calculation of 1 criterion - max"] the file: TehnSist_Krit1max
function [f,G] = TehnSist_Krit1max(x);
f=-(639+0.031*x(1)+0.0421*x(1).^2+0.039*x(2).^2);
G=[-(0.031+0.0421*2*x(1));
   -0.039*2*x(2)];
```

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```
% ["Restriction" Subprogramme] file:TehnSist_Const
function [c,ceq,DC,DCeq]=TehnSist_Const(x)
c(1)= (-506+0.71*x(1)+528*x(2)-1.9*x(2).^2)-21000;
c(2)=-(-506+0.71*x(1)+528*x(2)-1.9*x(2).^2)+10000;
DC=[0.71,           -0.071;
  (528-1.9*2*x(2)), -(528-1.9*2*x(2))];
 ceq=[];  DCeq=[];
```

In square brackets output parameters in the form of a point of an optimum of x1max and size of criterion function of f1max in this point are specified.

As a result of the solution of the problem (29), (33), (34) function (35) we will receive a point of an optimum of X_1^* ={80.0 48.5479} and the size of criterion of f_1^* =-1003.4 in this point. Similarly by other criteria:

$f_1(X)\rightarrow max$ X_1^* ={x_1=80.0, x_2=48.5479}, $f_1^* = f_1(X_1^*)$ =-1003.4;

$f_2(X)\rightarrow min$ X_2^* ={x_1=12.7076, x_2=21.55}, $f_2^* = f_2(X_2^*)$ =1000;

$f_3(X)\rightarrow max$ X_3^* ={x_1=80.0, x_2= 49.4079}, $f_3^* = f_3(X_3^*)$=-850.0912;

$f_4(X)\rightarrow min$ X_4^* ={x_1=15.7512, x_2=49.5421}, $f_4^* = f_4(X_4^*)$=-836.7402.

In fig. 1 the admissible set of points S formed by restrictions (34), points of private optimum is shown: X_1^*, X_2^*, X_3^*, X_4^*; points of auxiliary optimum: X_{12}^o, X_{13}^o, X_{34}^o, X_{42}^o. All these points are united in a contour which presents to $S^o \subset S$ - a set of points, optimum across Pareto.

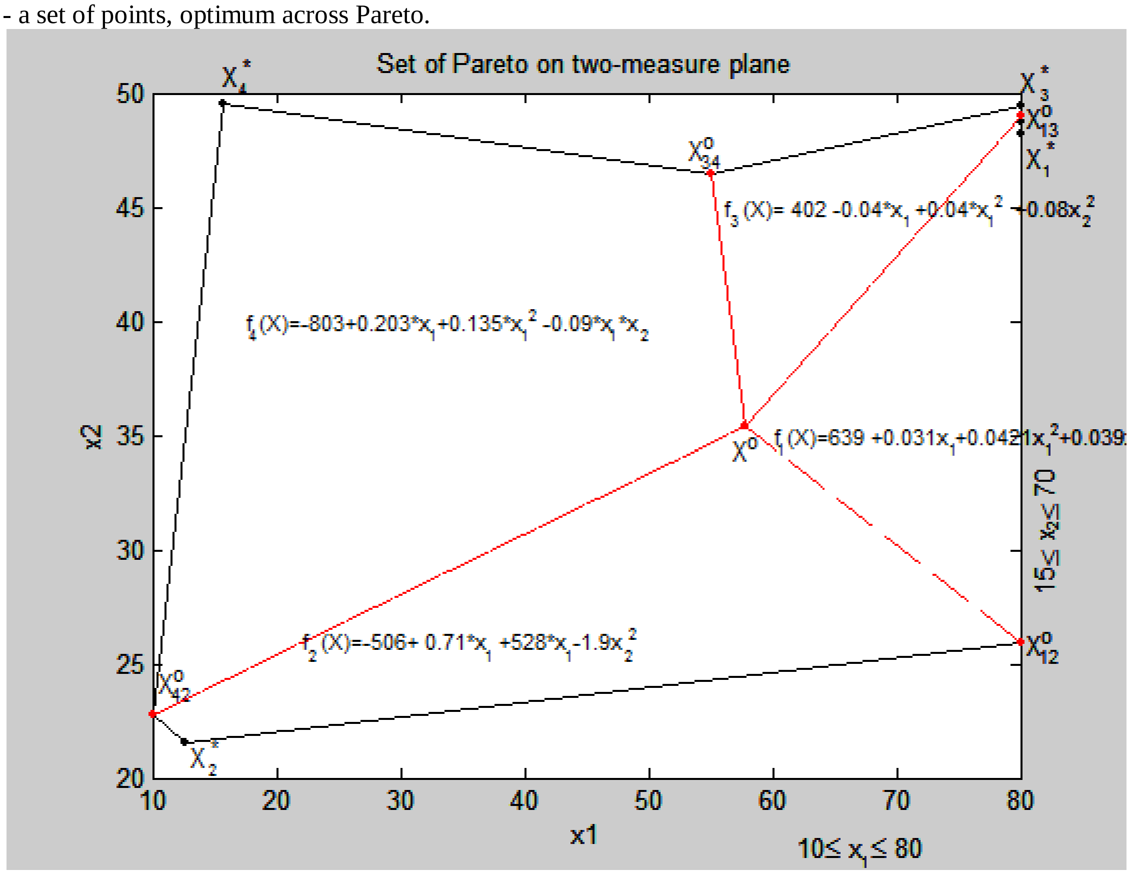

Fig. 1. An admissible set of points S tasks (29)-(34) and Pareto's great number, $S^o \subset S$ in two-dimensional system of coordinates of x_1 and x_2.

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Step 2. The worst size (anti-optimum) of each criterion on an admissible set of S is defined. For example, for the first criterion the appeal to the problem (29), (33), (34) looks as follows:

[x1min, f1min] =fmincon ('TehnSist_Krit1min', Xo, A, b, Aeq, beq, lb, ub, 'TehnSist_Const', options) (36)

where 'TehnSist_Krit1min' – the subprogramme of definition of function (the first criterion) and its gradient, $\frac{\partial f_k(X)}{\partial X}$, $k=\overline{1,4}$ on a Fortran.

```
% [The subprogramme "Calculation of 1 criterion - min"] the file: TehnSist_Krit1min
 function [f,G] = TehnSist_Krit1min(x);
f = (639+0.031*x(1)+0.0421*x(1).^2+0.039*x(2).^2);
G = [(0.031+0.0421*2*x(1));
      0.039*2*x(2)];
```

As a result for each $k=\overline{1,4}$ we receive points of private anti-optimum and size of criteria in them $f_k^o = f_k(X_k^o)$:

$f_1(X)\rightarrow min$ X_1^o ={x_1=10.0, x_2=21.5564}, f_1^o =f_1(X_1^o)=661.64;

$f_2(X)\rightarrow max$ X_2^o ={x_1=30.0543, x_2=49.5122}, f_2^o =f_2(X_2^o)=-21000.0;

$f_3(X)\rightarrow min$ X_3^o ={x_1= 10.0, x_2=21.5564}, f_3^o =f_3(X_3^o)= 442.7743;

$f_4(X)\rightarrow max$ X_4^o ={x_1=80.0, x_2=45.4499}, f_4^* = f_4(X_4^*)=250.1992.

Step 3. The analysis of a set of points, optimum across Pareto, for this purpose in optimum points of $X^*=\{X_k^*, k=\overline{1,4}\}$ are defined sizes of criterion functions of $F(X^*)=\{f_q(X_k^*), q=\overline{1,K}, k=\overline{1,K}\}$ and relative estimates

$\lambda(X^*)=\{\lambda_q(X_k^*), q=\overline{1,K}, k=\overline{1,K}\}$, $\lambda_k(X)=\frac{f_k(X)-f_k^o}{f_k^*-f_k^o}$, $\forall k\in K$, $d_k = f_k^* - f_k^o$:

$$F(X^*)=\begin{vmatrix} 1003 & 20759 & 845 & -274 \\ 664 & 9999 & 445 & -803 \\ 1006 & 21000 & 850 & -279 \\ 746 & 21000 & 608 & -837 \end{vmatrix},$$

$d1$ = 341.7786, $d2$ = -11001, $d3$ = 407.32, $d4$ = -586.54

$$\lambda(X^*)=\begin{vmatrix} 1.0000 & 0.0219 & 0.9864 & 0.0399 \\ 0.0078 & 1.0000 & 0.0057 & 0.9433 \\ 1.0079 & 0 & 1.0000 & 0.0486 \\ 0.2458 & 0 & 0.4048 & 1.0000 \end{vmatrix}.$$

From matrix $\lambda(X^*)$ follows that in private optimum points relative estimates reach the greatest sizes and are equal to unit.

Step 4. Creation of λ-problem.

Maximine problem of optimization with the normalized criteria:

$\lambda^o = \max_x \min_k \lambda_k(X)$, $G(X)\le 0$, $X\ge 0$,

it will be transformed to a standard problem of mathematical programming (λ-problem):

$\lambda^o = \max \lambda$, (37)

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$$\lambda - \frac{639 + 0.031x_1 + 0.0421x_1^2 + 0.039x_2^2 - f_1^o}{f_1^* - f_1^o} \le 0, \qquad (38)$$

$$\lambda - \frac{402 - 0.04x_1 + 0.04x_1^2 + 0.08x_2^2 - f_2^o}{f_2^* - f_2^o} \le 0, \qquad (39)$$

$$\lambda - \frac{-506 + 0.71x_1 + 528x_2 - 0.19x_2^2 - f_3^o}{f_3^* - f_3^o} \le 0, \qquad (40)$$

$$\lambda - \frac{-803.2 + 0.203x_1 + 0.135x_1^2 - 0.09x_1x_2 - f_4^o}{f_4^* - f_4^o} \le 0, \qquad (41)$$

$$10000 \le f_2(x) \le 21000; \quad 0 \le \lambda \le 1, \ 10 \le x_1 \le 80, \ 15 \le x_2 \le 70, \qquad (42)$$

where the vector of unknown has dimension of $N+1$: $X=\{x_1, x_2, \lambda\}$.

As a result of the solution of VPMP (29)-(34) at equivalent criteria and to λ-problem corresponding to it (37)-(42) we will receive:

• $X^o=\{X^o, \lambda^o\}=\{x_1=57.7806$, $x_2= 35.4081, \lambda^o=-0.4683\}$ - an optimum point which represents design parameters of the TS and the maximum relative assessment of $\lambda^o=0.4683$;

• $f_k(X^o)$, $k=\overline{1,K}$ - sizes of criteria (TS characteristics) $f_k(X^o)=\{-830 \quad 15848 \quad -634 \quad -525\}$;

• $\lambda_k(X^o)$, $k=\overline{1,K}$ - sizes of relative estimates of $\lambda_k(X^o)= \{0.4933 \quad 0.4683 \quad 0.4683 \quad 0.4683\}$;

• $\lambda^o=0.4683$ is the maximum bottom level among all relative estimates, measured in relative units: $\lambda^o=\min(\lambda_1(X^o), \lambda_2(X^o), \lambda_3(X^o), \lambda_4(X^o)) =0.4683$, λ^o – also call the guaranteed result in relative units, i.e. $\lambda_k(X^o)$ and according to the characteristic of the $f_k(X^o)$ TS it is impossible to improve, without worsening thus other characteristics.

The received point of X^o is shown in fig. 1. We will show interrelation of all relative estimates (criteria) from a point of X_2^* in three-dimensional space $\{x_1, x_2, \lambda\}$ in fig. 2.

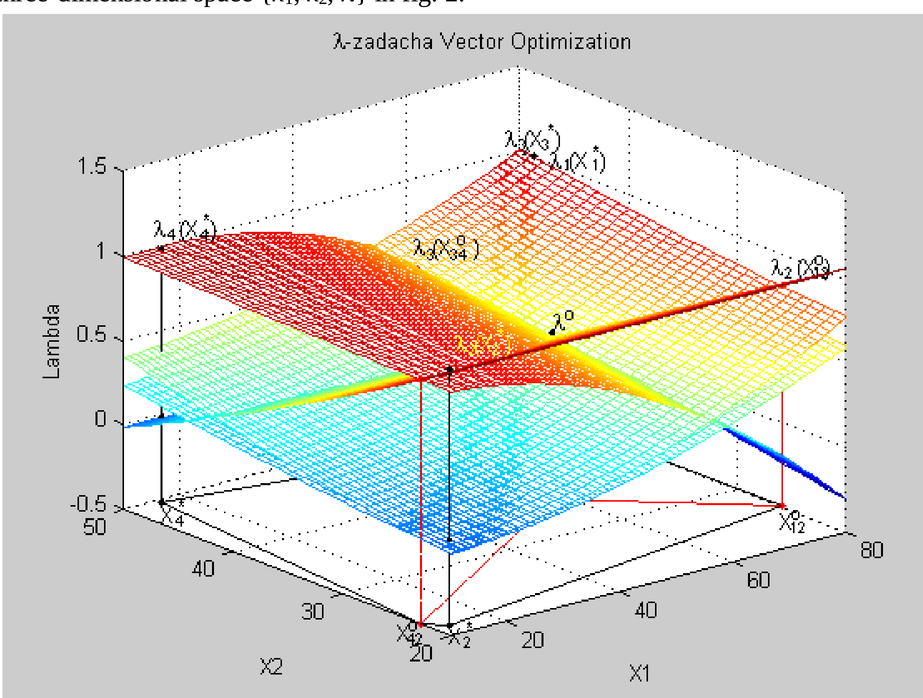

Fig. 2. λ-problem and points of an optimum of VPMP (29)-(34) in three-dimensional system of coordinates of x_1, x_2, λ.

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The point of an optimum of X^o (conditionally the center of a set of points, optimum across Pareto) characterizes, on the one hand, optimum parameters of the TS X^o, and about other, guaranteed result of λ^o=0.4683. In fig. 2 look interrelation of all criteria (TS characteristics), measured in relative units.

Conclusions

Thus, in work the methodology of optimization of parameters of difficult technical system on some set of functional characteristics that is one of the most important tasks of the system analysis and design is offered. The technology of creation of mathematical model of such system in the form of a vector problem and adoption of the optimum decision is presented. For the solution of this problem the methods based on normalization of criteria and the principle of guaranteed result are used. Results of the decision are a basis for decision-making on studied technical system. This methodology can be used at research, modeling and adoption of the optimum decision for a wide class of technical and other tasks.

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